## Principles for Designing Symmetric Protein Assemblies



Todd Yeates -- RosettaCon 2012

## Biological Protein Assemblies: An inspiration and a challenge


virus capsid


Heat shock protein

clathrin



CsoS4 or CcmL
proteins

bacterial microcompartments
Design assembly successes trail those using DNA

## Symmetry-Centric Approaches to Designing Protein Assemblies

## Biological Protein Assemblies

- Nearly always symmetric
- Predicted as early as 1956 by Crick and Watson
- Repetitive symmetric assemblies require fewer distinct contact or interface types

NATURE
March 10, 1956
Vol. 177

STRUCTURE OF SMALL VIRUSES
molecular level, a structure of a definite size and shape has to be built up from smaller units; namely, that the packing arrangements are likely to be repeated again and again-and hence that the subunits are likely to be related by symmetry elements.

Table 1. The Three Possible Cubic Point groups for a Spherical

| Crystallographic description | No. and type of rotation axes present | No. of asymmetric units | Platonie solid with these sym. metry elements |
| :---: | :---: | :---: | :---: |
| 23 | $\begin{aligned} & 3 \text { 2-fold } \\ & 43 \text {-fold } \end{aligned}$ | 12 | Tetrahedron |
| 432 | 6 4 4 3 3 3 -fold 4 | 24 | Cube Octahedron |
| 532 | 152 -fold <br> 103 -fold <br> 0 5-fold | 60 | Dodecahedron Icosahedron |

The number of sub-units will be the same as, or a multiple of, the
number of asymmetric units
F. H. C. Crick
J. D. Watson*

Medical Research Council Unit for the
Study of the Molecular Structure of
Biological Systems,
Cavendish Laboratory,
Cambridge
Jan. 23.

## Number of Distinct Contact Types as a Central Idea

- Limited Outcomes with Only a Single Distinct Contact Type:

$$
\begin{array}{cc}
\ldots 66666 \ldots & 6^{\circ} . . . \\
\text { linear or helical filaments } & 0_{0}^{9}
\end{array}
$$

- Richer Outcomes Using >1 Contact Type:



## Key Design Questions and a Connection to Group Theory

- What kinds of higher symmetries should be targeted for design, and how?
- How many contacts and in what geometries are required for various symmetries?
- Equivalent to a (incompletely solved) problem in group theory: What is the fewest number of elements of a (potentially infinite) group from which the group can be generated?


## A Connection Between Design and Group Theory

Number of designed contacts required

Minimum number of elements required to 'generate' the group

Examples:

$$
\begin{aligned}
& \{1, i,-1,-i\} \\
& \left\{\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]\left[\begin{array}{ll}
0 & 1 \\
-1 & 0
\end{array}\right]\left[\begin{array}{cc}
-1 & 0 \\
0 & -1
\end{array}\right]\left[\begin{array}{cc}
0 & -1 \\
1 & 0
\end{array}\right]\right]\left[\left\{\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]\left[\begin{array}{ll}
1 & 0 \\
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\end{array}\right]\left[\begin{array}{cc}
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\end{array}\right]\right]\right.
\end{aligned}
$$

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$$

## A Connection Between Design and Group Theory

Number of designed contacts required

Minimum number of elements required to 'generate' the group

Examples:


## A Brief Diversion:

## The space group preference problem




Top 1: ~33\% Bottom 55: 20\%


One of the most puzzling (and overlooked) problems in structural biology. The differences in probability span more than 2 orders of magnitude, yet there are no obvious energetic explanations.

Different Crystal Space Group Symmetries Have a Different Minimum Contact Number, C

This number relates to how easy it is (how many degrees of freedom there are) to build

p2, $\mathrm{C}=3$


$$
\mathrm{p} 4, \mathrm{C}=2
$$

$C$ is a property of the mathematical group, not the molecule

# Different Crystal Space Group Symmetries Have a Different Minimum Contact Number, C 

 This number relates to how easy it is (how many degrees of freedom there are) to build

p2mm, C=4

When the values of C for the 65 biological space groups were enumerated, they provided a powerful explanation for observed space group preferences.

## Agreement between the dimensionality for forming different space groups and their observed frequencies

- The 65 possible space group symmetries fall into 4 categories of increasing likelihood: $D=4,5,6,7 \quad$ (factor of ~8 for each increment in $D$ )

- Only one space group, $\mathrm{P} 2_{1} 2_{1} 2_{1}$, which dominates in macromolecular crystals, has D=7 !!
- A dimensionality analysis explains most of the observed phenomenon.


## Extending the Theory: Mirror image proteins provide a potentially powerful solution to the protein crystallization problem



Predictions from theory

- Proteins will crystallize much more easily if they can be prepared as a racemic mixture; this requires chemical synthesis of the mirror image protein (i.e. from Damino acids)
- P1(bar) will dominate for racemic crystallization of proteins; this highly specific prediction provides a powerful test of the theoretical ideas



Space group


Yeates and Kent (2012). Annu. Rev. Biophys

# Returning to the Problem of Designed Assembly: 

A remarkable number of highly symmetric groups can be generated with just two operators (or contact types)!

cubic symmetry octahedral symmetry formed only by two-fold and four-fold symmetric contacts

icosahedral symmetry formed only by two-fold and three-fold symmetric contacts

## Design Rules (2-fold + 3-fold)

|  | Symmetry | Construction | Geometry of s | try elements |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| cages | Cages and shells |  |  |  | Tetrahedral, T |
|  | T | Dimer-Trimer | $54.7^{\circ}$, | Intersecting | 2-fold \& 3-fold |
|  | 0 | Dimer-Trimer | $35.3{ }^{\circ}$, | Intersecting | Intersecting |
|  | I | Dimer-Trimer | $20.9{ }^{\circ}$, | Intersecting |  |
|  | Double-layer rings |  |  |  |  |
|  | $\mathrm{D}_{\text {n }}$ | Dimer-Dimer | $180 \%$, | Intersecting |  |



## filaments and rods

## Extension of the Symmetric Contact Idea to a Strategy for Designing Self-Assembling Protein Materials

- Natural oligomeric (e.g. dimeric and trimeric) proteins can serve as the building blocks
- Fusing two such proteins together (e.g. by genetic engineering) provides the two interactions needed for a rich variety of designs



## The Geometry of the Symmetry Axes Dictates the Assembly and Must be Controlled:

Two example outcomes:


Regularly ordered self-assembly results when the relative orientation of the symmetry elements matches one of the known point, layer, or space groups.

A General Method for Designing
Self-Assembling Protein Materials

- Fusion of two simple oligomers (e.g. dimer + trimer)
- Use of a
continuous $\alpha$-helix to dictate geometry
- Satisfies predictability req., though not freely designable. (combinatorial)


Padilla, Colovos, \& Yeates, PNAS, 98, 2217 (2001)

## 1st Design

- Intended architecture: tetrahedral cage (T), 12 subunits, $170 \AA$ diameter, $1 / 2 \mathrm{MDa}$
- Components:
- bromoperoxidase, 276 aa (trimer)
- a helical linker (9 residues from L9 ribosomal protein)
- influenza M1 coat protein, 150 aa (dimer)
- Symmetry element geometry:
- angle between 2-fold and 3-fold: $53.2^{\circ}$ [ideal $\left.=54.7^{\circ}\right]$
- failure to intersect: $2.8 \AA$ [ideal $=0.0$ ]
- Expressed and purified in soluble form from E. coli (48 kD)


Padilla, Colovos, \& Yeates, PNAS, 98, 2217 (2001)

## 1st Design - A partial success




Discrete particles of approximately the right size, but polymorphic. Crystals never obtained!

## Original Design Revisited... 11 years later: Two mutations



Gln24 was mutated to valine to attract the leucine on the linker

## Original Design Revisited... 11 years later: Two mutations



## A first atomic structure of a designed protein cage

( $\sim 11$ years after publication of idea and preliminary experiments)


- 12 subunits
- Pseudo-tetrahedral symmetry
- Partially flattened (crystal packing and weak helical linker)
Lai, et al. (2012). Science 336, 1129.


## Three independent cages in two crystal forms



70 Å diameter (hypothetical) inner sphere

## Surprisingly large deviations from symmetric design (~8 Å)

## Distorted helices

## Distorted dimeric interfaces



Not surprising in retrospect. Interface polymorphism was revealed after initial design choice. (Luo, et al.)

New results by others in symmetry-based design of complex protein assemblies/materials:
A variation on the oligomer fusion strategy
Sinclair JC, Davies KM, Vénien-Bryan C, Noble ME. (2011). Nat. Nanotechnol. 6, 558-62.


C4+C2
multiple geometrically tolerant connections at a shared symmetry axis


D4+D2 with a shared symmetry axis

## New results by others in symmetry-based design of complex protein assemblies/materials:

## A variation on the oligomer fusion strategy



Yeates, TO. (2011) Nat.
Nanotechnol. 6, 541-2
Sinclair JC, Davies KM, Vénien-Bryan C, Noble ME. (2011). Nat. Nanotechnol. 6, 558-62.


# New results by others in symmetry-based design of complex protein assemblies/materials: 

## Metal interface design



Brodin, J.D., et al. (2012) Nat Chem 4, 375-382

New results by others in symmetry-based design of complex protein assemblies/materials:
Introducing new interface(s) by sequence design

or

fusion


New results by others in symmetry-based design of complex protein assemblies/materials:

Introducing new interface(s) by sequence design

## Design of a protein crystal based on coiled coil motifs

(a)

(b)


Lanci, C.J., et al. (2012) PNAS 109, 7304-7309

New results by others in symmetry-based design of complex protein assemblies/materials:
Introducing new interface(s) by sequence design

## Design of protein cages and cubic assemblies based on 'general' oligomers and sequence design



King, N.P., et al. (2012). Science 336, 1171-1174.

## Future Directions

- Algorithm/strategy improvements; success rates remain low
- Theoretical enumeration of complete rules and possible outcomes
- Biomedical and nanotechnology applications
- display (e.g. vaccine), containment/delivery, bioactive (e.g. enzymatic) solids and surfaces

Possible combinations of two symmetry point groups and their assembly outcomes


## Principles for Designing Symmetric Protein Assemblies



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